

# Motion Induced by Light: Photokinetic Effects in the Rayleigh Limit

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Structured beams of light can move small objects in surprising ways. Particularly striking examples include observations of polarization-dependent forces acting on optically isotropic objects and tractor beams that can pull objects opposite to the direction of the light's propagation. Here we develop a theoretical framework in which these effects vanish at the leading order of light scattering theory. Exotic optical forces emerge instead from interference between different orders of multipole scattering. These effects create a rich variety of ways to manipulate small objects with light, so-called photokinetic effects. Applying this formalism to the particular case of Bessel beams offers useful insights into the nature of tractor beams and the interplay between spin and orbital angular momentum in vector beams of light, including a manifestation of orbital-to-spin conversion.

Beams of light exert forces and torques on illuminated objects. The resulting photokinetic effects have attracted considerable interest because of their role in optical micromanipulation [1]. Gradients in the light's intensity give rise to induced-dipole forces that are responsible for optical trapping [2]. Scattering and absorption give rise to radiation pressure through transfer of the light's linear momentum density. The momentum density, in turn, is steered by gradients in the wavefronts' phase [3] and also by the curl of the light's spin angular momentum density [4–6]. The repertoire of effects governed by the amplitude, phase and polarization profiles of a structured beam of light features several surprises. Circularly polarized beams of light, for example, influence optically isotropic objects in ways that linearly polarized light cannot [6, 7]. Appropriately structured beams of light can even transport small objects upstream against the direction of propagation [8–12], and are known as tractor beams by reference to the science fiction trope [13].

Here we present a theory for photokinetic effects in vector beams of light that lends itself to interpretation of experimental results. It naturally distinguishes strong and easily observed first-order effects from more subtle second-order effects, and provides a basis for designing new modes of light for optical micromanipulation. Our formulation focuses primarily on objects that are substantially smaller than the wavelength of light, the so-called Rayleigh regime, for which we obtain analytical results. This analysis explains observation of spin-dependent forces acting on isotropic objects [6], sets limits on propagation-invariant tractor beams [10–12], and predicts the existence of as-yet unobserved effects such as orbital-to-spin conversion in helical Bessel beams. The theory is useful therefore for developing and optimizing optical micromanipulation tools such as first-order tractor beams [9] and knotted force fields [14].

## DESCRIPTION OF A BEAM OF LIGHT

The vector potential of a monochromatic beam of light of frequency  $\omega$  may be decomposed into Cartesian components as

$$\mathbf{A}(\mathbf{r}, t) = \sum_{j=0}^2 a_j(\mathbf{r}) e^{i\varphi_j(\mathbf{r})} e^{-i\omega t} \hat{\mathbf{e}}_j \quad (1)$$

where  $a_j(\mathbf{r})$  is the real-valued amplitude of the beam along direction  $\hat{\mathbf{e}}_j$  at position  $\mathbf{r}$  and  $\varphi_j(\mathbf{r})$  is the associated phase. This unusual factorization is useful because it expresses photokinetic effects in terms of quantities that can be controlled experimentally.

For vector potentials that satisfy the Coulomb gauge condition,

$$\nabla \cdot \mathbf{A}(\mathbf{r}) = 0, \quad (2)$$

the electric and magnetic fields are related to the vector potential by

$$\mathbf{E}(\mathbf{r}, t) = -\partial_t \mathbf{A}(\mathbf{r}, t) = i\omega \mathbf{A}(\mathbf{r}, t) \quad \text{and} \quad (3a)$$

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t). \quad (3b)$$

The amplitudes of the vector potential's components contribute to the light's intensity through

$$I(\mathbf{r}) = \frac{1}{2\mu c} |\mathbf{E}(\mathbf{r}, t)|^2 = \frac{\omega^2}{2\mu c} \sum_{j=0}^2 a_j^2(\mathbf{r}). \quad (4)$$

With this, the local polarization may be written as

$$\hat{\mathbf{e}}(\mathbf{r}) = \frac{2\mu c}{\omega^2 I(\mathbf{r})} \sum_{j=0}^2 a_j(\mathbf{r}) e^{i\varphi_j(\mathbf{r})} \hat{\mathbf{e}}_j. \quad (5)$$

This description provides a point of departure for discussing light's properties and its ability to exert forces on small objects.

## PROPERTIES OF A BEAM OF LIGHT

The time-averaged linear momentum density carried by the beam of light is given by Poynting's theorem,

$$\mathbf{g}(\mathbf{r}) = \frac{1}{2\mu c^2} \Re \{ \mathbf{E}^*(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t) \}. \quad (6)$$

where  $c$  is the speed of light in the medium. Expressing  $\mathbf{g}(\mathbf{r})$  in terms of experimentally controlled quantities,

$$\mathbf{g}(\mathbf{r}) = \frac{\omega}{2\mu c^2} \sum_{j=0}^2 a_j^2(\mathbf{r}) \nabla \varphi_j(\mathbf{r}) + \frac{1}{2} \nabla \times \mathbf{s}(\mathbf{r}), \quad (7)$$

reveals that the light's momentum is guided both by phase gradients [3, 6] and also by the curl of the time-averaged spin angular momentum density [4–6, 15]

$$\mathbf{s}(\mathbf{r}) = \frac{\omega}{2\mu c^2} \Im \{ \mathbf{A}^*(\mathbf{r}, t) \times \mathbf{A}(\mathbf{r}, t) \} \quad (8a)$$

$$= \frac{i}{\omega c} I(\mathbf{r}) \hat{\epsilon}(\mathbf{r}) \times \hat{\epsilon}^*(\mathbf{r}). \quad (8b)$$

A beam of light also can carry orbital angular momentum [4, 16, 17],

$$\boldsymbol{\ell}(\mathbf{r}) = i \frac{\omega}{2\mu c^2} I(\mathbf{r}) \sum_{j=0}^2 \epsilon_j(\mathbf{r}) [\mathbf{r} \times \nabla \epsilon_j^*(\mathbf{r})] \quad (9a)$$

$$= \frac{\omega^2}{2\mu c} \sum_{j=0}^2 a_j^2(\mathbf{r}) [\mathbf{r} \times \nabla \varphi_j(\mathbf{r})], \quad (9b)$$

which is distinct from its spin angular momentum. Equation (9) suggests little connection between  $\boldsymbol{\ell}(\mathbf{r})$  and  $\mathbf{s}(\mathbf{r})$ . A direct connection has been noted, however, in experimental studies of particles' circulation in circularly polarized optical traps [6, 7, 18] and has been dubbed spin-to-orbital conversion. The converse process by which the orbital angular momentum content of a beam imbues the light with spin angular momentum has received less attention.

## OPTICAL FORCES IN THE RAYLEIGH REGIME

### First-order photokinetic effects

Optical forces arise from the transfer of momentum from the beam of light to objects that scatter or absorb the light. They are not simply proportional to  $\mathbf{g}(\mathbf{r})$ , however, but rather arise from more subtle mechanisms. To illustrate this point, we consider the force exerted by a beam of light on an object that is smaller than the wavelength of light. Taking this limit permits us to adopt the Rayleigh approximation that the light's instantaneous electric field is uniform across the illuminated object's

volume. The electromagnetic field then induces electric and magnetic dipole moments in the particle,

$$\mathbf{p}(\mathbf{r}, t) = \alpha_e \mathbf{E}(\mathbf{r}, t) \quad \text{and} \quad (10a)$$

$$\mathbf{m}(\mathbf{r}, t) = \alpha_m \mathbf{B}(\mathbf{r}, t), \quad (10b)$$

respectively, where  $\alpha_e = \alpha'_e + i\alpha''_e$  is the particle's complex electric polarizability and  $\alpha_m$  is the corresponding magnetic polarizability.

The induced electric dipole moment responds to the light's electromagnetic fields through the time-averaged Lorentz force [19]

$$\mathbf{F}_e(\mathbf{r}) = \frac{1}{2} \Re \{ (\mathbf{p}(\mathbf{r}, t) \cdot \nabla) \mathbf{E}^*(\mathbf{r}, t) \} \quad (11)$$

$$+ \frac{1}{2} \Re \{ \partial_t \mathbf{p}(\mathbf{r}, t) \times \mathbf{B}^*(\mathbf{r}, t) \},$$

$$= \frac{1}{2} \Re \left\{ \alpha_e \sum_{j=0}^2 E_j(\mathbf{r}, t) \nabla E_j^*(\mathbf{r}, t) \right\}. \quad (12)$$

Expressed in terms of experimentally controlled parameters, the electric dipole contribution to the time-averaged force is

$$\mathbf{F}_e(\mathbf{r}) = \frac{1}{2} \mu c \alpha'_e \nabla I(\mathbf{r}) + \mu c \alpha''_e \sum_{j=0}^2 a_j^2(\mathbf{r}) \nabla \varphi_j(\mathbf{r}). \quad (13)$$

The first term in Eq. (13) is the manifestly conservative intensity-gradient force that is responsible for trapping by single-beam optical traps [2]. The second describes a non-conservative force [3, 15] that is proportional to the phase-gradient contribution to  $\mathbf{g}(\mathbf{r})$ . This is the radiation pressure experienced by a small particle and is responsible for the transfer of orbital angular momentum from helical modes of light [20]. Because  $\alpha''_e > 0$  for scattering by conventional materials, radiation pressure tends to drive illuminated objects downstream along the direction of the light's propagation.

Surprisingly, the spin-curl contribution to  $\mathbf{g}(\mathbf{r})$  has no counterpart in  $\mathbf{F}_e(\mathbf{r})$ , this missing term having been canceled [15] by the induced dipole's coupling to the magnetic field in Eq. (11). This means that the radiation pressure experienced by a small particle is not simply proportional to the light's momentum density [5, 15], as might reasonably have been assumed. The absence of any spin-dependent contribution may be appreciated because  $\mathbf{s}(\mathbf{r})$  involves off-diagonal terms in the components of the polarization whereas  $\mathbf{F}_e(\mathbf{r})$  does not [15]. Compelling experimental evidence for spin-dependent forces acting on isotropic objects [6] therefore cannot be explained as a first-order photokinetic effect.

The corresponding result for magnetic dipole scattering has a form [21] analogous to Eq. (13),

$$\mathbf{F}_m(\mathbf{r}) = \frac{1}{2} \Re \left\{ \alpha_m \sum_{j=0}^2 B_j(\mathbf{r}, t) \nabla B_j^*(\mathbf{r}, t) \right\}. \quad (14)$$

This contribution to the total optical force similarly is comprised of a conservative intensity-gradient term and a non-conservative phase-gradient term, with no contribution from the light's spin angular momentum density.

At the dipole order of light scattering, the photokinetic forces on optically isotropic scatterers are governed by intensity gradients and phase gradients, and are essentially independent of the light's polarization. Higher multipole moments [10] similarly contribute to the conservative intensity-gradient force and to the radiation pressure. None of these terms, however, feature contributions from the light's spin angular momentum.

Equation (13) also constrains designs for tractor beams. A beam propagating along  $\hat{z}$  with  $\mathbf{g}(\mathbf{r}) \cdot \hat{z} > 0$  would exert a retrograde force if  $\mathbf{F}_e(\mathbf{r}) \cdot \hat{z} < 0$ . In a propagation-invariant beam satisfying  $\partial_z I(\mathbf{r}) = 0$ , we expect  $\sum_{j=0}^2 a_j(\mathbf{r}) \partial_z \varphi_j(\mathbf{r}) > 0$ . The phase-gradient term in Eq. (13) therefore has a positive axial projection for conventional materials with  $\alpha_e'' > 0$ . This means that propagation-invariant beams cannot act as tractor beams for induced electric dipoles. A similar line of reasoning yields an equivalent result for  $\mathbf{F}_m(\mathbf{r})$ . The apparent absence of propagation-invariant tractor beams at the dipole order of multipole scattering is consistent with numerical studies [10, 11] that find no pulling forces for particles much smaller than the wavelength of light.

### Second-order photokinetic effects

Exotic effects such as polarization-dependent forces and tractor beam action are restored by the interference between the scattered fields. For the particular case of the electric and magnetic dipole fields, such interference leads to a force of the form [10, 21]

$$\begin{aligned} \mathbf{F}_{em}(\mathbf{r}) &= -\frac{k^4}{12\pi c \epsilon_0} \Re \{ \mathbf{p}^*(\mathbf{r}, t) \times \mathbf{m}(\mathbf{r}, t) \} \\ &= \frac{k^3 \mu}{12\pi \epsilon_0} \left[ -\Im \{ \alpha_e^* \alpha_m \} \nabla I(\mathbf{r}) \right. \\ &\quad \left. + \epsilon_0 \omega^2 \Im \{ \alpha_e^* \alpha_m \} \Re \{ [\mathbf{A}^*(\mathbf{r}, t) \cdot \nabla] \mathbf{A}(\mathbf{r}, t) \} \right. \\ &\quad \left. - 2\omega \Re \{ \alpha_e^* \alpha_m \} \mathbf{g}(\mathbf{r}) \right]. \end{aligned} \quad (15)$$

The first term in Eq. (16) contributes to the intensity-gradient trapping force. The second describes a non-conservative force that appears not to have been discussed previously. It is influenced by spatial variations in the light's polarization but is symmetric under exchange of the components' indexes and so does not depend on the spin angular momentum density. The third is proportional to the linear momentum density and therefore includes both phase-gradient and spin-curl contributions.

Interference between electric and magnetic dipole scattering therefore can account for spin-dependent forces of

the kind observed in optical trapping experiments [6, 7, 18, 22]. It can be shown, moreover, that spin-dependent contributions to optical forces appear at higher orders of multipole scattering due to interference between the fields scattered by induced electric and magnetic multipoles. Higher-order effects than those captured by Eq. (16) may be accentuated in particles larger than the wavelength of light, particularly through Mie resonances.

Equation (16) also suggests a mechanism by which propagation-invariant modes can act as tractor beams. The first two terms of Eq. (16) are directed substantially transverse to  $\mathbf{g}(\mathbf{r})$ , and so cannot contribute to retrograde forces. The prefactor of the third term is negative for conventional materials, and thus inherently describes a pulling force. The overall axial force can be directed upstream for objects that satisfy

$$\Re \{ \alpha_e^* \alpha_m \} > \frac{\mu^2}{k^2} \Im \left\{ c \alpha_e + \frac{1}{c} \alpha_m \right\}. \quad (17)$$

This condition is not generally met for particles smaller than the wavelength of light, which is why tractor-beam action generally is anticipated for larger particles whose Mie resonances favor forward scattering [10, 11, 23].

### APPLICATION TO BESSEL BEAMS

As an application of this formalism, we consider the forces exerted by a monochromatic Bessel beam [24, 25], whose vector potential may be written in cylindrical coordinates,  $\mathbf{r} = (\rho, \theta, z)$ , as

$$\mathbf{A}_{m,\alpha}(\mathbf{r}, t) = a_0 e^{-i\omega t} \hat{\mathbf{P}} J_m(\tilde{\rho}) e^{im\theta + i \cos \alpha k z}. \quad (18a)$$

Here,  $a_0$  is the wave's amplitude,  $k = \omega/c$  is the wavenumber of an equivalent plane wave at frequency  $\omega$ ,  $\tilde{\rho} = \sin \alpha k \rho$  is a dimensionless radial coordinate, and the operator  $\hat{\mathbf{P}}$  projects the scalar wavefunction into a vector field satisfying the Coulomb gauge condition, Eq. (2). The projection operator [26],

$$\hat{\mathbf{P}}_{\phi}^{\text{TE}} = -\frac{1}{k} \nabla \times \hat{z}, \quad (18b)$$

describes a transverse electric Bessel beam that is azimuthally polarized for  $m = 0$ . Using operator notation not only yields compact expressions for the vector Bessel beams, but also clarifies the symmetries of their wave functions.

Bessel beams are characterized by a convergence angle  $\alpha$  that reduces the axial component of the momentum density relative to that of a plane wave. An object that scatters light into the forward direction might thereby increase the momentum density in the beam, and so would have to recoil upstream to conserve momentum. This is the basis for the proposal [8, 10, 11] that a Bessel beam can act as a tractor beam.

The vector potential in Eq. (18) also is characterized by an integer winding number,  $m$ , that governs the helical pitch of the beam's wavefronts and endows the light with orbital angular momentum [17, 27]. This is expressed in the axial component of the orbital angular momentum density,

$$\ell(\mathbf{r}) \cdot \hat{z} = m a_0^2 \frac{\omega}{2\mu c^2} J_m^2(\tilde{\rho}), \quad (19)$$

which is proportional to  $m$ .

Remarkably, this Bessel beam's spin angular momentum density,

$$\mathbf{s}(\mathbf{r}) = m a_0^2 \frac{1}{2\mu\omega} \frac{1}{\rho} \frac{dJ_m^2(\tilde{\rho})}{d\rho} \hat{z}, \quad (20)$$

also is proportional to the helical winding number  $m$ . Equation (20) therefore describes orbital-to-spin angular momentum conversion, an effect that appears not to have been described previously. Having wavefront helicity control the light's degree of spin polarization complements spin-to-orbital conversion [7] in which a beam's state of polarization contributes to its wavefronts' helicity. Like spin-to-orbital conversion, orbital-to-spin conversion vanishes in the paraxial limit ( $\alpha \rightarrow 0$ ).

In considering the forces exerted by Bessel beams, it is convenient to replace the dipole polarizabilities by the first-order Mie scattering coefficients,  $a_1$  and  $b_1$ , which are dimensionless and tend to be of the same order of magnitude for small particles. They are related to the polarizabilities by  $\alpha_e = i6\pi\epsilon_0 n_m^2 a_1/k^3$  and  $\alpha_m = i6\pi b_1/(\mu k^3)$  [10, 28]. With this substitution, the net optical force due to dipole scattering,  $\mathbf{F}(\mathbf{r}) = \mathbf{F}_e(\mathbf{r}) + \mathbf{F}_m(\mathbf{r}) + \mathbf{F}_{em}(\mathbf{r})$ , has an axial component

$$\begin{aligned} \frac{\mu F_z(\mathbf{r})}{3\pi k^2 a_0^2 \cos \alpha \sin^2 \alpha} &= \Re \{a_1 + \cos^2 \alpha b_1\} f_{m,\alpha}^2(\tilde{\rho}) \\ &+ \Re \{b_1\} \sin^2 \alpha J_m^2(\tilde{\rho}) - \Re \{a_1^* b_1\} f_{m,\alpha}^2(\tilde{\rho}), \end{aligned} \quad (21)$$

where

$$f_{m,\alpha}^2(\tilde{\rho}) = \left[ \frac{dJ_m(\tilde{\rho})}{d\tilde{\rho}} \right]^2 + \left[ \frac{J_m(\tilde{\rho})}{\tilde{\rho}} \right]^2. \quad (22)$$

The first two terms in Eq. (21) arise from electric and magnetic dipole scattering, respectively, and tend to push the scatterer along the  $+\hat{z}$  direction. The third term, which arises from interference, generally is negative and describes a pulling force. The Bessel beam acts as a tractor beam only if this second-order term dominates the first-order terms. This condition is met by scatterers whose polarizabilities satisfy Eq. (17). Similar results can be obtained for the radially polarized Bessel beam. On the basis of these considerations, we conclude that a Bessel beam only acts as a tractor beam for Rayleigh particles under exceptional circumstances. The formulation of first- and second-order optical forces reveals that this

limitation is a generic feature of propagation-invariant beams of light and is not specific to Bessel beams.

Distinguishing first- and second-order photokinetic effects is useful for assessing the nature of the forces exerted by beams of light on Rayleigh particles. Trapping by optical tweezers and the torque exerted by transfer of orbital angular momentum are examples of first-order effects. Second-order effects include the spin-curl force, which has been studied experimentally [6] and observed numerically [29], but has not previously been explained theoretically [5, 15]. The pulling force exerted by propagation-invariant tractor beams, including Bessel beams, similarly turns out to be a second-order effect.

Orbital-to-spin conversion similarly emerges as a property of helical Bessel beams and is the counterpart to spin-to-orbital conversion, which has been widely discussed [7, 18]. Beyond revealing subtle properties of vector light waves, this formalism also provides a basis for developing new modes of optical micromanipulation. Whereas tractor beams with continuous propagation invariance are inherently limited in their ability to transport small objects, Eq. (13) provides guidance for designing more powerful and longer-range [30] tractor beams with discrete propagation invariance that can operate at dipole order. Solenoidal waves [9] appear to be an example of such first-order tractor beams. The theory of photokinetic effects suggests how to optimize such modes and how to discover new examples.

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- $$\hat{\mathbf{P}}_{\phi}^{\text{TM}} = -\frac{1}{k^2} \nabla \times \nabla \times \hat{\mathbf{z}}. \quad (23)$$
- The Bessel beam described by Eq. (1) in Ref. [10] may be obtained with a linear combination of  $\hat{\mathbf{P}}_{\phi}^{\text{TE}}$  and  $\hat{\mathbf{P}}_{\phi}^{\text{TM}}$ .
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